YEAR 1 | VECTORS 1

VECTOR OPERATIONS | KEY FACTS

- Given two vectors, $\mathbf{p}=\begin{pmatrix}3\\5\end{pmatrix}$ and $\mathbf{q}=\begin{pmatrix}-2\\4\end{pmatrix}$, we can perform the following vector operations:
    - e.g. $\mathbf{p}+\mathbf{q}=\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}-2\\4\end{pmatrix}=\begin{pmatrix}3+(-2)\\5+4\end{pmatrix}=\begin{pmatrix}1\\9\end{pmatrix}$ (vector *addition/subtraction*)
    - e.g. $\mathbf{p}=8\begin{pmatrix}3\\5\end{pmatrix}=\begin{pmatrix}8\times3\\8\times5\end{pmatrix}=\begin{pmatrix}24\\40\end{pmatrix}$ (scalar *multiplication*.)
- Vector multiplication is covered on the A-Level Further Mathematics course.

EQUAL AND PARALLEL VECTORS | KEY FACTS

- Two vectors are *equal* if they have the *same magnitude and direction* (they do not need to occupy the same position in space).
- If $\mathbf{a}$ and $\mathbf{b}$ are *parallel*, they can be written as $\mathbf{b}=t\mathbf{a}$, for some scalar t.
- A *unit vector* is a vector with a magnitude of one unit. To find a unit vector in the direction of a particular vector, divide the vector by its magnitude.

PARALLEL VECTORS | EXAMPLE-PROBLEM PAIR

Given that $\mathbf{a}=\begin{pmatrix}1\\2\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}-3\\p\end{pmatrix}$, and $\mathbf{c}=\begin{pmatrix}5\\10\end{pmatrix}$.

(a) Show that $\mathbf{a}$ is parallel to $\mathbf{c}$.

(b) Find the value of p such that $\mathbf{b}$ is parallel to $\boldsymbol{a}$.

Given that $\mathbf{p}=2\mathbf{i}-3\mathbf{j}$, $\mathbf{q}=5\mathbf{i}-7.5\mathbf{j}$ and $\mathbf{r}=m\mathbf{i}+12\mathbf{j}$

(a) Show that $\mathbf{p}$ is parallel to $\mathbf{q}$.

(b) Find the value of m such that $\mathbf{p}$ is parallel to $\mathbf{r}$.

Find two unit vectors parallel to $\begin{pmatrix}3\\4\end{pmatrix}$.

Find the unit vector in the direction of $\mathbf{q}=5\mathbf{i}-12\mathbf{j}$.

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Displacement and Position Vectors | Key Facts

- A vector that represents the *displacement between two points* is called a *displacement vector*.
- A vector that represents the *displacement from a fixed origin* is called a *position vector*.
- If points A and B have *position vectors* $\mathbf{a}$ and $\mathbf{b}$, then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Displacement and Position Vectors | Example-Problem Pairs

(a) Find the distance between points A and B with position vectors $\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$.

(b) Point C has position vector $c = \begin{pmatrix} 2 \\ p \end{pmatrix}$.

Find the exact value of p such that $AC = 3AB$.

Points P and Q have position vectors $\overrightarrow{OP} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Find the distance PQ.

Midpoints | Key Facts

- The midpoint of the line segment connecting with position vectors $\boldsymbol{a}$ and $\mathbf{b}$ is given by $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$.

Points on a Straight Line | Key Facts

- If points A, B and C lie on a straight line, the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ must be parallel.
  - Therefore $\overrightarrow{AB} = k\overrightarrow{AC}$
  - If points lie on a straight line, they are <u>collinear</u>.
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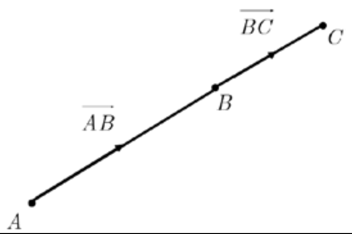

Parallelograms and Rhombi | Key Facts

- If $\overrightarrow{AB} = \overrightarrow{DC}$ then $ABCD$ is a parallelogram.
- If in addition, $|\overrightarrow{AB}| = |\overrightarrow{BC}|$ then $ABCD$ is a rhombus.